MATHEMATICS CLASS 12

HOLIDAY HOMEWORK

1.DO NCERT with example first.

2.Revise all chapters tought

3.Some more assignments has been given on website.Solve them .

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CBSE 12th Mathematics
Chapter 2 (Inverse Trigonometric Functions)
Important Questions Unsolved

SECTION - A

Question number 1 to 19 carry 1 mark each.

Q. 1: What is the principal value of:

$$sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)?$$

Q.2: Write the principle value of:

$$cos^{-1}\left(\frac{1}{2}\right) - 2\,sin^{-1}\left(-\frac{1}{2}\right).$$

Q.3: Using principal value, evaluate the following:

$$cos^{-1}\left(cos\,\frac{2\pi}{3}\right) + sin^{-1}\left(sin\,\frac{2\pi}{3}\right).$$

Q.4: Write the principal value of:

$$\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$$

Q.5: Write the principal value of $sec^{-1}(-2)$.

Q.6: What is the principle value of:

$$cos^{-1}\left(cos\frac{2\pi}{3}\right) + sin^{-1}\left(sin\frac{2\pi}{3}\right)?$$

Q.7: Find the principle value of :

$$\tan^{-1}\sqrt{3} - \sec^{-1}(-2).$$

Q.8: Write the principal value of

$$\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3}).$$

Q.9: Write the value of

$$\tan^{-1}\left[\left(2sin^{-1}\frac{\sqrt{3}}{2}\right)\right]$$

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Q.10: If

$$tan^{-1}x + tan^{-1}y = \frac{\pi}{4}, xy < 1,$$

Then write the value of $x + y + xy$.

Q. 11: Evaluate:

$$\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$$

Q.12: Using principal value, evaluate the following:

$$\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$$

Q.13: What is the principal value of:

$$sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)?$$

Q.14: Write the value of

$$\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$$

Q.15: Write the principle value of:

$$cos^{-1}\left(\frac{1}{2}\right) - 2\,sin^{-1}\left(-\frac{1}{2}\right)$$

Q.16: Write the principal value of

$$tan^{-1}(1) + cos^{-1}\left(-\frac{1}{2}\right).$$

Q. 17: Write the value of $tan\left(2tan^{-1}\frac{1}{5}\right)$

Q. 18: If $\sin\left(sin^{-1}\frac{1}{5} + cos^{-1}x\right) = 1$

then find the value of x.

Q.19: Solve for x:

$$2\,tan^{-1}(cos\,x) = tan^{-1}(2\,cosec\,x)$$

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SECTION – B

Question number 20 to 53 carry 4 mark each.

Q.20: Prove the following:

$$tan^{-1}\frac{1}{3}+tan^{-1}\frac{1}{5}+tan^{-1}\frac{1}{7}+tan^{-1}\frac{1}{8}=\frac{\pi}{4}.$$

Q.21: Prove that:

$$\sin^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{16}{65}\right)=\frac{\pi}{2}$$

Q.22: Solve for x:

$$\tan^{-1}3x+\tan^{-1}2x=\frac{\pi}{4}.$$

Q.23: Prove that:

$$\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,\frac{-1}{\sqrt{2}}\leq x\leq 1.$$

Q.24: If

$$\tan^{-1}\left(\frac{x-2}{x-4}\right)+\tan^{-1}\left(\frac{x+2}{x+4}\right)=\frac{\pi}{4},\text{ find the value of } x.$$

Q.25: Solve for x:

$$tan^{-1}(x+1)+tan^{-1}(x-1)=tan^{-1}\frac{8}{31}$$

Q.26: Prove the following:

$$cot^{-1}\left(\frac{xy+1}{xy-1}\right)+cot^{-1}\left(\frac{yz+1}{y-z}\right)+cot^{-1}\left(\frac{zx+1}{z-x}\right)=0$$

$$(0<xy,yz,zx<1)$$

Q.27: Show that:

$$\tan\left(\frac{1}{2}sin^{-1}\frac{3}{4}\right)=\frac{4-\sqrt{7}}{3}$$

Q.28: Solve the following equation:

$$\cos\left(tan^{-1}x\right)=\sin\left(cot^{-1}\frac{3}{4}\right).$$

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Q.29: Prove the following:

$$cos\left(sin^{-1}\frac{3}{5}+cot^{-1}\frac{3}{2}\right)=\frac{6}{5\sqrt{13}}$$

Q.30: If

$$\tan^{-1}\frac{x-3}{x-4}+\tan^{-1}\frac{x+3}{x+4}=\frac{\pi}{4},$$

Then find the value of x.

Q.31: Prove the following:

$$tan^{-1}\left(\frac{1}{3}\right)+tan^{-1}\left(\frac{1}{5}\right)+tan^{-1}\left(\frac{1}{7}\right)+tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$$

Q.32: Solve for x:

$$\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}.$$

Q.33: Prove the following:

$$cot^{-1}\left(\frac{\sqrt{1+sinx}+\sqrt{1-sin\,x}}{\sqrt{1+sinx}-\sqrt{1-sin\,x}}\right)=\frac{x}{2},x\,\epsilon\left(0,\frac{\pi}{4}\right)$$

Q.34: Solve for x:

$$2\,tan^{-1}(cos\,x)=tan^{-1}(2\,cosec\,x).$$

Q.35: Prove the following:

$$tan^{-1}x+tan^{-1}\left(\frac{2x}{1-x^2}\right)=tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right).$$

Q.36: Prove the following:

$$cos\,[tan^{-1}\{sin(cot^{-1}x)\}]=\sqrt{\frac{1+x^2}{2+x^2}}.$$

Q.37: Prove that:

$$tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]=\frac{\pi}{4}-\frac{1}{2}\,cos^{-1}x,-\frac{1}{\sqrt{2}}\leq x\leq 1.$$

Q.38: Solve the equation for x

$$\sin^{-1}x+\sin^{-1}(1-x)=\cos^{-1}x$$

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Q.39: If

$$\cos^{-1}\frac{x}{a}+\cos^{-1}\frac{y}{b}=\alpha,$$

Prove that

$$\frac{x^2}{a^2}-2\frac{xy}{ab}\cos\alpha+\frac{y^2}{b^2}=\sin^2\alpha.$$

Q.40: Prove that

$$\tan\left\{\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}=\frac{2b}{a}$$

Q.41: Prove the following:

$$\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right),x\in(0,1)$$

Q.42: Prove the following:

$$\cos^{-1}\left(\frac{12}{13}\right)+\sin^{1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{56}{65}\right)$$

Q.43: Prove the following:

$$cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]=\frac{x}{2},x\in\left(0,\frac{\pi}{4}\right)$$

Q.44: Find the value of:

$$\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)$$

Q.45: Prove that:

$$tan^{-1}\left(\frac{cosx}{1+sinx}\right)=\frac{\pi}{4}-\frac{x}{2},x\epsilon\left(-\frac{\pi}{2},\frac{\pi}{2}\right).$$

Q.46: Prove that

$$sin^{-1}\left(\frac{8}{17}\right)+sin^{-1}\left(\frac{3}{5}\right)=cos^{-1}\left(\frac{36}{85}\right).$$

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Q.47: Find the value of the following:

$$\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right],$$

$|x|<1, y>0$ *and* $xy<1$.

Q.48: Prove that:

$$tan^{-1}\left(\frac{1}{2}\right)+tan^{-1}\left(\frac{1}{5}\right)+tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}.$$

Q. 49: Prove that

$$cot^{-1}\left(\frac{\sqrt{1+sin\,x}+\sqrt{1-sin\,x}}{\sqrt{1+sin\,x}-\sqrt{1-sin\,x}}\right)=\frac{x}{2}; x\,\epsilon\left(0,\frac{\pi}{4}\right).$$

Q.50: Prove that

$$2\,tan^{-1}\left(\frac{1}{5}\right)+sec^{-1}\left(\frac{5\sqrt{2}}{7}\right)+2\,tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}.$$

Q.51: If $\sin[\cot^{-1}(x+1)]=\cos(\tan^{-1}x)$, thenfind x.

Q.52: If

$$(tan^{-1}x)^2+(cot^{-1}x)^2=\frac{5\pi^2}{8}, \text{then find } x.$$

Q.53: Prove that:

$$tan^{-1}\frac{1}{5}+tan^{-1}\frac{1}{7}+\ tan^{-1}\frac{1}{3}+tan^{-1}\frac{1}{8}=\frac{\pi}{4}$$

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6. If $\begin{bmatrix} x+3 & 4 \\ y-4 & x+y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$, find x and y. *(C.B.S.E. 2008)*

7. Construct a 2×2 matrix whose element a_{ij} is given by $a_{ij} = i + 2j$. *(C.B.S.E. 2008)*

8. Construct a 2×2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = \frac{i}{j}$ *(C.B.S.E. 2008 C)*

9. If $\begin{bmatrix} x+2y & -y \\ 3x & 4 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 6 & 4 \end{bmatrix}$, find the values of x and y. *(C.B.S.E. 2008 C)*

10. If matrix $A = [1 \ \ 2 \ \ 3]$, write AA' where A' is the transpose of matrix A. *(C.B.S.E. 2009)*

11. Write the value of the determinant

$$\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$$

(C.B.S.E. 2009)

12. If A is an invertible matrix of order 3 and $|A| = 5$, then find $|\text{adj } A|$. *(C.B.S.E. 2009, 2011C)*

13. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then find k if $|2A| = k|A|$. *(C.B.S.E. 2009)*

14. Write the value of the determinant $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$. *(C.B.S.E. 2009)*

15. If $A = (a_{ij}) = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}$

and $B = (b_{ij}) = \begin{bmatrix} 2 & 1 & -1 \\ -3 & 4 & 4 \\ 1 & 5 & 2 \end{bmatrix}$, find $a_{22} + b_{21}$. *(C.B.S.E. 2009)*

16. If $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 7 & 11 \\ k & 23 \end{pmatrix}$, find the value of k. *(C.B.S.E. 2010)*

17. If $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$, then for what value of α is A an identity matrix? *(C.B.S.E. 2010)*

18. Write A^{-1} for $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$. *(C.B.S.E. 2011)*

19. If A is an invertible matrix of order 3 and $|A| = 5$, find the value of $|\text{adj } A|$. *(C.B.S.E. 2011C)*

20. If $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, write the minor of the element a_{23}. *(C.B.S.E. 2012)*

Group B (4 marks)

21. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, prove that $A - A^T$ is a skew symmetric matrix where A^T denotes the transpose of A.

(C.B.S.E. 2001)

Revision Exercise 3(b)
(From Board Papers)

Group A (1 mark)

1. Find the values of x and y if

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

(C.B.S.E. 2008)

2. Find the cofactor of a_{12} in the following :

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

(C.B.S.E. 2008)

3. Evaluate : $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

(C.B.S.E. 2008)

4. For what value of x, is the following matrix singular?

$$\begin{bmatrix} 3-2x & x+1 \\ 2 & 4 \end{bmatrix}$$

(C.B.S.E. 2008)

5. A matrix A of order 3×3 has determinant 4. Find the value of $|3A|$.

(C.B.S.E. 2008)

22. If $A = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix}$, show that $A + A^T$ is a symmetric matrix where A^T denotes the transpose of matrix A. *(C.B.S.E. 2001)*

23. If $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and $B = [-2 \quad -1 \quad -4]$, verify that $(AB)' = B'A'$. *(C.B.S.E. 2002)*

24. Construct a 2×3 marix A, whose elements are given by $a_{ij} = \frac{(i-2j)^2}{2}$. *(C.B.S.E. 2002)*

25. From the following equation, find the values of x and y :

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$$ *(C.B.S.E. 2002 C)*

26. Using the properties of determinants, evaluate the following : $\begin{vmatrix} 0 & ab^2 & ac^2 \\ a^2b & 0 & bc^2 \\ a^2c & cb^2 & 0 \end{vmatrix}$ *(C.B.S.E. 2003)*

Using the properties of determinants, show that (27 – 28) :

27. $\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a+x+y+z)$. *(C.B.S.E. 2003)*

28. $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ y+z & z+x & x+y \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$. *(C.B.S.E. 2003, 2005, 2008)*

29. Find X such that X. $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$. *(C.B.S.E. 2003)*

30. Solve for x and y given that

$$\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$. *(C.B.S.E. 2003 C)*

31. Express $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. *(C.B.S.E. 2004)*

32. Using the properties of determinants, solve for x

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0.$$ *(C.B.S.E. 2004, 2005)*

33. If $A = \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$, find $f(a)$ where $f(x) = x^2 - 5x + 7$. *(C.B.S.E. 2004 C)*

34. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, prove that $A^3 - 4A^2 + A = O$ *(C.B.S.E 2005)*

35. Show that $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$ where a, b and c are in A.P. *(C.B.S.E. 2005)*

36. If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, find k so that $A^2 = 8A + kI$. *(C.B.S.E. 2005 C)*

37. Using the properties of determinants, prove that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c).$$ *(C.B.S.E. 2005 C)*

38. Express the matrix $\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. *(C.B.S.E. 2006)*

39. Using the properties of determinants, prove that

$$\begin{vmatrix} 3a & -a+b & -a+c \\ a-b & 3b & c-b \\ a-c & b-c & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca).$$ *(C.B.S.E. 2006)*

40. If a, b and c are in A.P., show that

$$\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0.$$ *(C.B.S.E. 2006)*

41. Find the value of x if

$$\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O.$$ *(C.B.S.E. 2006 C)*

42. Using the properties of determinants, prove that

$$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}.$$ *(C.B.S.E. 2006 C)*

43. If $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$, show that $A^2 - 6A + 17I = O$. *(C.B.S.E. 2007)*

Hence, find A^{-1}.

Using the properties of determinants, prove that **(44 – 47) :**

44. $$\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx).$$ *(C.B.S.E. 2007)*

45. $$\begin{vmatrix} 1 & bc & bc(b+c) \\ 1 & ca & ca(c+a) \\ 1 & ab & ab(a+b) \end{vmatrix} = 0.$$ *(C.B.S.E. 2007)*

46. $$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3.$$ *(C.B.S.E. 2008)*

47. $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$. *(C.B.S.E. 2008 C, 2012)*

48. If x, y, z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, show that $xyz = -1$. *(C.B.S.E. 2008 C)*

Using the properties of determinants, prove each of the following (49 – 55) :

49. $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$. *(C.B.S.E. 2008 C, 2012)*

50. $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$. *(C.B.S.E. 2008 C)*

51. $\begin{vmatrix} x & x^2 & 1+ax^3 \\ y & y^2 & 1+ay^3 \\ z & z^2 & 1+az^3 \end{vmatrix} = (1+axyz)(x-y)(y-z)(z-x)$. *(C.B.S.E. 2008 C)*

52. $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$. *(C.B.S.E. 2009)*

53. $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3+b^3+c^3-3abc$. *(C.B.S.E. 2009)*

54. $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = 1$. *(C.B.S.E. 2009)*

55. $\begin{vmatrix} a^2+1 & ab & ac \\ ba & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$. *(C.B.S.E. 2009, 2011C)*

56. Using elementary row transformations, find the inverse of the matrix $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$. *(C.B.S.E. 2010)*

Using properties of determinants, prove that (57 – 60) :

57. $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$. *(C.B.S.E. 2011, 2011C)*

58. $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$. *(C.B.S.E. 2012)*

59. $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$. *(C.B.S.E. 2012)*

60. $\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a+x+y+z)$. *(C.B.S.E. 2012)*

Group C (6 marks)

Using matrix method, solve each of the following system of equations (61 – 66) :

61. $x - y + z = 3$
$2x + y - z = 2$
$-x - 2y + 2z = 1$ *(C.B.S.E. 2001)*

62. $x + y + z = 6$
$x + 2y + 3z = 14$
$x + 4y + 7z = 30$ *(C.B.S.E. 2001 C)*

63. $x + 2y - 3z = -4$
$2x + 3y + 2z = 2$
$3x - 3y - 4z = 11$ *(C.B.S.E. 2002 C, 2008)*

64. $5x + 3y + z = 16$
$2x + y + 3z = 19$
$x + 2y + 4z = 25$ *(C.B.S.E. 2002 C)*

65. $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$
$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$
$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$ *(C.B.S.E. 2002 C)*

66. $2x + 6y = 2$
$3x - z = -8$
$2x - y + z = -3$ *(C.B.S.E. 2003)*

67. Given that $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, find AB.

Use this to solve the following system of equations :

$$\begin{aligned} x - y &= 3 \\ 2x + 3y + 4z &= 17 \\ y + 2z &= 7 \end{aligned}$$

(C.B.S.E. 2003 C)

Using matrices, solve each of the following system of equations (68 – 71) :

68. $2x - 3y + z = -1$
$x - 2y + 3z = 6$
$-3y + 2z = 0$ *(C.B.S.E. 2004 C)*

69. $x + y + z = 4$
$2x - y + z = -1$
$2x + y - 3z = -9$ *(C.B.S.E. 2005)*

70. $x + y - z = 1$
$x - y - z = 1$
$3x + y - 2z = 3$ *(C.B.S.E. 2005)*

71. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}.

Use it to solve the following system of equations :

$$\begin{aligned} 2x - 3y + 5z &= 16 \\ 3x + 2y - 4z &= -4 \\ x + y - 2z &= -3 \end{aligned}$$

(C.B.S.E. 2005 C)

Using matrices, solve the following system of equations (72 – 77) :

72. $3x - y + z = 5$
$2x - 2y + 3z = 7$
$x + y - z = -1$ *(C.B.S.E. 2006)*

73. $3x + 4y + 2z = 8$
$2y - 3z = 3$
$x - 2y + 6z = -2$ *(C.B.S.E. 2006 C)*

74. $x + y + z = 6$
$x - y + z = 2$
$2x + y - z = 1$ *(C.B.S.E. 2007)*

75. $x + 2y + z = 1$
$x + 3z = 11$
$2x - 3y = 1$ *(C.B.S.E. 2007)*

76. $x + 3y + 4z = 8$
$2x + y + 2z = 5$
$5x + y + z = 7$ *(C.B.S.E. 2008)*

77. $2x - y + z = 3$
$-x + 2y - z = -4$
$x - y + 2z = 1$ *(C.B.S.E. 2008)*

Using elementary transformations, find the inverse of each of the following matrices (78 – 80) :

78. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$ *(C.B.S.E. 2008)*

79. $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ *(C.B.S.E. 2008)*

80. $\begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$ *(C.B.S.E. 2008)*

81. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}.

Using A^{-1}, solve the system of equations

$$\begin{aligned} 2x - 3y + 5z &= 11 \\ 3x + 2y - 4z &= -5 \\ x + y - 2z &= -3 \end{aligned}$$

(C.B.S.E. 2008 C, 2009)

Using matrices, solve each of the following system of equations (82 – 83) :

82. $x + 2y + z = 7$
$x + 3z = 11$
$2x - 3y = 1$ *(C.B.S.E. 2008 C)*

83. $x + y + z = 6$
$x + 2z = 7$
$3x + y + z = 12$ *(C.B.S.E. 2009)*

84. Obtain the inverse of the following matrix using elementary operations :

$$A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}.$$

(C.B.S.E. 2009)

85. Using properties of determinants, show that

$$\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc\,(a+b+c)^3 .$$

(C.B.S.E. 2010)

86. Using matrix method, solve the system of equations

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$, $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$, $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$, $x, y, z \neq 0$.

(C.B.S.E. 2011)

87. Using elementary row transformations, find the inverse of matrix

$$\begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}.$$

(C.B.S.E. 2011)

88. If $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 2 & 0 & -3 \end{bmatrix}$, find A^{-1} and hence solve the system of equations

$x - 2y + z = 0, \quad -y + z = -2, \quad 2x - 3z = 10.$

(C.B.S.E. 2011C)

89. Using matries, solve the system of equations

$$\begin{aligned} x - y + 2z &= 7 \\ 3x + 4y - 5z &= -5 \\ 2x - y - 3z &= 12 \end{aligned}$$

(C.B.S.E. 2012)

90. Using elementary operations, find the inverse of matrix

$$\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}.$$

(C.B.S.E. 2012)

ANSWERS

1. $x = 3$ and $y = 3$ **2.** 46 **3.** $a^2 + b^2 + c^2 + d^2$ **4.** 1

5. 108 **6.** $x = 2$ and $y = 7$ **7.** $\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$ **8.** $\begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}$

9. $x = 2, y = -3$ **10.** (14) **11.** 0 **12.** 25

13. $k = 4$ **14.** 0 **15.** 1 **16.** $k = 17$

17. $\alpha = 0$ **18.** $\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$ **19.** 6 **20.** 7

24. $\begin{bmatrix} \frac{1}{2} & \frac{9}{2} & \frac{25}{2} \\ 0 & 2 & 8 \end{bmatrix}$ **25.** $x = 2$ and $y = 9$ **26.** $2a^3 b^3 c^3$ **29.** $\begin{bmatrix} -60 & -142 \\ 25 & 59 \end{bmatrix}$

30. $x = 2$ and $y = 1$ **31.** $\frac{1}{2}\begin{bmatrix} 6 & -3 \\ -3 & -2 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$ **32.** 0 or $3a$

33. $\begin{bmatrix} -15 & -20 \\ 20 & 15 \end{bmatrix}$ **36.** -7 **38.** $\frac{1}{2}\begin{bmatrix} 2 & -3 & 1 \\ -3 & 16 & 9 \\ 1 & 9 & 10 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 9 & 9 \\ -9 & 0 & -3 \\ -9 & 3 & 0 \end{bmatrix}$

41. $-2, -14$ **43.** $\frac{1}{17}\begin{bmatrix} 4 & 3 \\ -3 & 2 \end{bmatrix}$ **56.** $\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$ **61.** $x = \frac{5}{3}, y = -\frac{4}{3} + k, z = k$

62. $x = -2 + k, y = 8 - 2k, z = k$ **63.** $x = 3, y = -2, z = 1$ **64.** $x = 1, y = 2, z = 5$

65. $x = 2, y = 3, z = 5$ **66.** $x = -2, y = 1, z = 2$ **67.** $x = 2, y = -1, z = 4$

68. $x = 1, y = 2, z = 3$ **69.** $x = -1, y = 2, z = 3$ **70.** $x = 2, y = 1, z = 2$

71. $A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$; $x = 2, y = 1, z = 3$

72. $x = 1, y = -1, z = 1$

73. $x = -2, y = 3, z = 1$

74. $x = 1, y = 2, z = 3$

75. $x = -1, y = -1, z = 4$

76. $x = 1, y = 1, z = 1$

77. $x = 1, y = -2, z = -1$

78. $\begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$

79. $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$

80. $\begin{bmatrix} -2 & \frac{1}{2} & 1 \\ 11 & -1 & -6 \\ 4 & -\frac{1}{2} & -2 \end{bmatrix}$

81. $A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$; $x = 1, y = 2, z = 3$

82. $x = 2, y = 1, z = 3$

83. $x = 3, y = 1, z = 2$

84. $\begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix}$

86. $x = 2, y = 3$ and $z = 5$

87. $\begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix}$

88. $\begin{bmatrix} 3 & -6 & -1 \\ 2 & -5 & -1 \\ 2 & -4 & -1 \end{bmatrix}$, $x = 2,\ y = 0,\ z = -2$

89. $x = 2, y = 1$ and $z = 3$

90. $\begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$